Acoustic Attenuation in a Solid Propellant[1]

B. H. NALL[2]
Applied Physics Laboratory, Johns Hopkins University, Silver Spring, Md.

The acoustic attenuation in 1-in.-diam by 15-in.-long rods cast from a double-base propellant mixture has been measured for longitudinal excitation from 500 to 14,000 cps in discrete steps by the successive use of quarterwave (λ/4) resonance, half-wave (λ/2) resonance, and pulse techniques. The data are observed to give a reasonable fit to a Q of 5.5. No change in Q is observed for λ/4 resonance data taken over the range of temperatures 55° to 75°F, but the velocity of sound in the rod is found to be a sensitive decreasing function of the temperature, i.e., $\Delta v/\Delta t \approx 2.1\%$ of room temperature velocity/°F. The effect on λ/2 resonance of increasing the static pressure from 1 atm to 1000 psi is found to be small, with a slight decrease in Q and in velocity. Room temperature propellant sound velocity is about 810 m/sec, giving an effective Young's Modulus = 1.1×10^{10} d/cm^2.

A KNOWLEDGE of the acoustic attenuation in solid propellants is important because unstable combustion of solid propellants in rockets is accompanied by the excitation, sometimes intense, of acoustic resonances in the rocket system (1).[3] The solid propellant possesses, by virtue of its binder, viscoelastic properties (2,3); it may, therefore, be a rather lossy material for acoustic excitation.

An accepted method for determining the acoustic attenuation at audio frequencies in a self-supporting solid is to study its qualities as a resonant rod (4,5). However, it has been shown that in a resonant rod the attenuation acts to set a theoretical limit to the order of the highest harmonic that can be excited (5). It has also been shown that, in practice, in the case of longitudinal excitation, misalignment of the driving force axis with the rod axis as well as other displacement factors further limit the order of uppermost harmonic which can be generated (5). Therefore, in order to measure attenuation over the audio range in a relatively lossy material, one will need to supplement the resonant rod method with other loss measuring techniques.

This paper describes an experiment in which both the resonant rod method and pulse techniques are used in obtaining data on the attenuation in the audio frequency region for a cast double-base propellant. This, apparently, is the first report of a procedure for making longitudinal attenuation measurements in this frequency range on such a lossy material, more particularly a solid propellant, using a single rod specimen size.

Attenuation Parameters

The attenuation in a system of continuously distributed elements, e.g., in a transmission line or a long rod, can be expressed in terms of either of the related parameters, quality factor Q or attenuation coefficient α. The relationship, given in textbooks, e.g., Ref. 6, is

$$\alpha = \pi f/Qv = \pi/Q\lambda \qquad [1]$$

where f is the frequency, λ the wavelength, and v the phase velocity by definition equal to $f\lambda$. Often the simplest way to measure Q in an experiment is to use the relationship $Q = f_0/\Delta f$, where f_0 is the resonance frequency and Δf is the full width of the resonance curve at the half-power points. In a nonresonant system, α in nepers per unit length can be obtained from the usual expression for the amplitude A of a traveling sine wave at any distance z in a very long rod $A_z = A_0 e^{-\alpha z e j(\omega t - \beta z)}$, giving

$$\alpha = \log_e \frac{|A_1|/|A_2|}{z_2 - z_1} \qquad [2]$$

Experimental

Quarter Wave and Half Wave Resonant Rod Systems

Two 1-in.-diam rods, cast from the same "batch" of propellant mixture and machined to 15-in. length, were used in these experiments.[4] Preliminary experiments indicated that the Q's would be low ($\leqslant 10$), requiring a relatively large driving force. As a consequence, it was found that the power dissipation from driving currents flowing in a "voice coil" wound directly on the rod at the end would produce an unsafe rise in temperature, prohibiting use of the electrodynamic driving system. Instead, the method of Wegel and Walther (4), cementing a piece of magnetic earphone diaphragm to the rod end and using a bipolar earphone as a driver, was used. In a few cases, as an independent check of results on the λ/2 resonant rod, a similar earphone system was used at the opposite end as a detector. However, the data presented here were obtained on Sonotone 3T ceramic transducers.

In a rigid support system, one runs into problems with undesired support resonances that may cause ambiguities. The relative amplitudes of such resonances will depend upon the point and the direction of application of the driving force and upon the relative mechanical resistances or, in effect, the $(f_{\text{resonances}}/Q)$'s and the effective masses of the possible resonances. It is not practicable for the Q of each possible resonance in a support system to be controlled. The effective mass, however, can be made large. In order to obtain λ/4 rod resonance, a section of 3-in.-diam brass rod was cemented to the specimen rod end with a minimum of Duco cement. The brass section formed the basis for an essentially rigid support of the rod inside a plywood box, as shown in Fig. 1a. In the case of the λ/2 resonant rod system in Fig. 1b, both the plywood box and a heavy plate support at the center were more easily excited to resonance than was the rod. The transducer arrangement shown in Fig. 1b with transducer D_2 connected to transducer D_1 to make the signals in phase for λ/2 resonance turned out to be a satisfactory method for minimizing the effects of interfering resonances. Obviously the stylus are deflected in opposite directions for this resonance but are deflected in the same direction for simple

Received by ARS April 11, 1962.
[1] This work was supported by the Bureau of Naval Weapons, Department of the Navy, under Contract NOrd 7386.
[2] Engineer, Research Center.
[3] Numbers in parentheses indicate References at end of paper.

[4] The rods were supplied by Theos Angelus of the Allegany Ballistics Laboratory, Cumberland, Md.

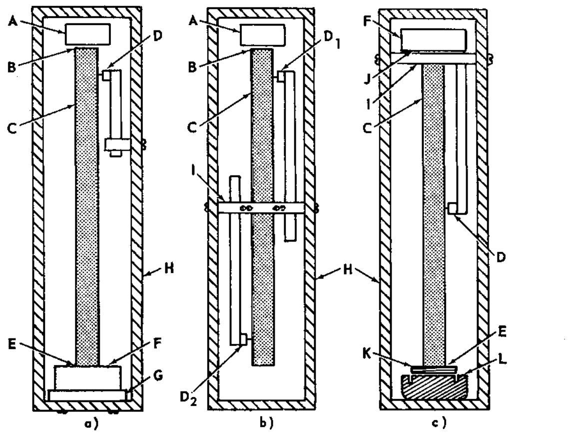

Fig. 1 Views of experimental arrangement of a) the $\lambda/4$ resonant rod system, b) the $\lambda/2$ resonant rod system, and c) the pulsed rod system. A is a bipolar magnet earphone; B is a magnetic diaphragm; C is the propellant rod; D is a Sonotone 3T ceramic cartridge supported by a $\frac{1}{8}$-in. Al rod; E is the Duco cement line; F is a section of 3-in. brass rod; G is acoustic insulating board; H is the plywood box; I is the $\frac{1}{2}$-in. steel plate support; J is a fiber shim; K is the driving coil wound on a section of a 2-in. Lucite rod; L is a permanent magnet

movement of the center plate I; thus, interfering signals are essentially cancelled. A block diagram of the instrumentation used for the $\lambda/4$ and $\lambda/2$ experiments is shown in Fig. 2. To obtain a resonance curve, one varies the frequency of the sine wave generator in incremental steps, recording the amplified output of the transducer D for each frequency. If the range of frequency is not too large, the driving force is, to a good approximation, constant.

As stated previously, the higher the attenuation in the rod, the fewer are the harmonic orders that can be excited. The difficulties encountered in getting the resonance curves for the first order in both the end-supported and the center-supported rods made use of another technique necessary for getting data at higher frequencies. However, two additional experiments were performed as follows.

Pressure and Temperature Effects

The $\lambda/4$ arrangement shown in Fig. 1a was used to get data on the effect of temperature on sound velocity and on Q for a limited range of temperature extending downward from 75°F.

The $\lambda/2$ experiment shown in Fig. 1b was placed in a tank in order to observe pressure effects from atmospheric pressure to 1000 psi.

Pulse Systems

Pulse techniques are more generally applied in the megacycle range (7), but attenuation in longitudinally excited rods has been studied previously using pulses (5). The magnetic drive system employed for resonant excitation was found to be unsuitable for pulse operation because of hysteresis and associated effects. Consequently, an electrodynamic drive, with the driver coil wound on a $\frac{1}{4}$-in. section of 2-in.-diam Lucite rod cemented to the propellant rod, was used for pulse excitation. End loading by such a Lucite section which would have added serious complications in the case of resonant excitation is not a problem in the case of a pulsed system. The brass termination F in Fig. 1c was not rigidly fastened to the plywood box but rather rested on fiber shims J, which in turn rested on the steel plate I. This arrangement gave the fewest spurious effects. A 2-in.-diam coil produced the maximum drive for the particular permanent magnet L used here.

It has been shown that at least three complete cyles are necessary in a sinusoidal pulse train to establish a definite excitation frequency (8). The experimental quantities needed for each frequency of interest are 1) the pulse amplitudes at two or more points along the rod and the distance of travel between those points, or 2) the amplitudes at a given point for a direct signal and for a 100% end-reflected signal and the distance traveled by the reflected signal. The data were conveniently recorded by using a Polaroid camera to photograph the series of events as displayed on the screen of an oscilloscope. The instrumentation is shown in Fig. 3.

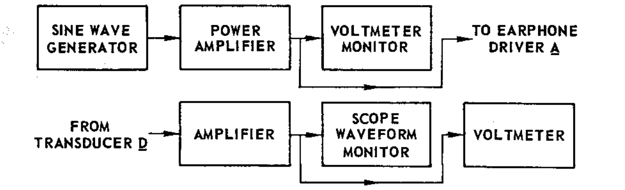

Fig. 2 Block diagram of instrumentation used with $\lambda/4$ and $\lambda/2$ resonant rod arrangements shown in Figs. 1a and 1b

Results

As a check on the general experimental approach used here, data on the attenuation in a polystyrene rod and in a Lucite rod were obtained by pulse techniques and compared with Q's obtained from rods in $n\lambda/2$ resonance ($n = 1,3,5, \ldots$). The two methods of determining attenuation were in essential agreement for both materials. The attenuation data in polystyrene corresponded to a Q of 200 and a velocity about 1800 m/sec. The attenuation data in Lucite were fitted by a Q of 30 and a velocity about 2080 m/sec.

The $\lambda/4$ resonance curve for the propellant was moderately symmetrical about $f_0 = 530$ cps, with $f_0/\Delta f = 6.3$. As was stated earlier, more difficulty was encountered in freeing the $\lambda/2$ resonance curve from spurious resonances in the support system, and the proper phasing of the transducers D_1 and D_2 in Fig. 1b greatly reduced the interference. Rod resonance was centered near 1000 cps and the $f_0/\Delta f$ measured about 5.5.

The use of pulse techniques on the propellant specimen was more complicated than for Lucite and polystyrene. In the pulse measurements on Lucite, a Q of 30 in a 0.93-m length permitted two round trips, or four passes of the pulse train, through the rod to be easily discernible at the transducer in an arrangement very similar to that shown in Fig. 1c for the propellant. Of course, a Q of 200 in a 0.82-m length of polystyrene permitted many more such passes to be observable. However, in the case of the propellant, adherence to the criterion of three or more complete cycles in the pulse train made the signal duration at frequencies below 5000 cps long enough to cause considerable overlap of direct and reflected signals at the transducer. Furthermore, the driving current pulse always appeared through stray coupling on to the scope display, thereby causing interference with the first cycle or so of the signal first passing the transducer. For these lower frequencies it was necessary to obtain the amplitude of the signal first passing the transducer by analysis of the composite wave train. At an intermediate range, 5000 to 8500

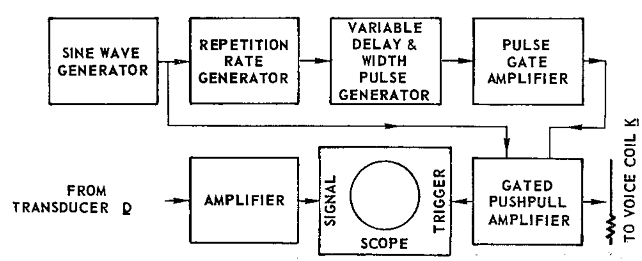

Fig. 3 Block diagram of pulse instrumentation used with the rod system shown in Fig. 1c

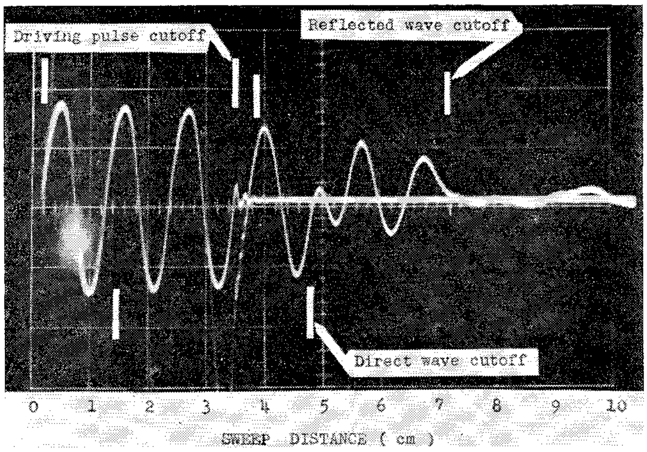

Fig. 4 Attenuation measurement at 4500 cps recorded by oscilloscope camera

cps, the amplitude of the direct signal and that of the reflected signal were measurable directly on the oscilloscope photograph. For higher frequencies the attenuation was so great that the reflected signal could not be observed. In this case the transducer was moved along the rod until the amplitude at a new point on the rod was a sensible fraction of the first point amplitude.

An example of an attenuation measurement using the pulse technique is shown in Fig. 4. The stray coupling previously mentioned rendered the signal useless until the driving pulse was cut off; therefore, as an accurate timing reference, the driving pulse was intentionally superposed on the attenuation data by double exposure. The following information is necessary for analyzing Fig. 4: rod length is 38.1 cm; transducer position from reflecting end is 19.6 cm; scope sweep time is 0.2 msec/cm; delay time in the transducer is 0.04 msec; zero offset of horizontal sweep is +0.15 cm. Fig. 4 is analyzed as follows:

1 In terms of sweep distance, the driving current pulse duration is approximately 3.35 cm, and cutoff occurs at 3.50 cm.

2 Arrival of cutoff edge of the reflected signal (second pass signal) at the transducer occurs at 7.20 cm with approximately 180° phase reversal occurring at the reflecting end for this signal.

3 From points 1 and 2, phase velocity is calculated to be 820 m/sec.

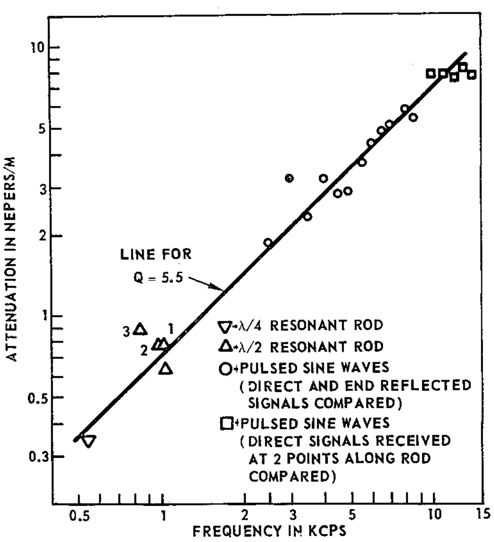

Fig. 5 Attenuation data, including an experiment on the effect of pressuring a λ/2 resonant rod to 1000 psi. Triangular points labeled 1, 3, 2 are the attenuation before, during, and after pressurization, respectively

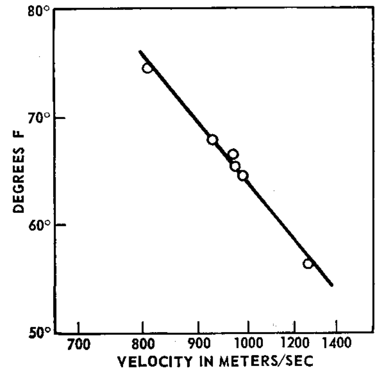

Fig. 6 Variation of longitudinal bar velocity with temperature for λ/4 resonance

4 Use of point 3 allows determination of the arrival at the transducer of the cutoff edge of the direct (first pass) signal. Again, in terms of sweep distances, arrival occurs at approximately 4.80 cm.

5 Therefore, for the interval 3.85 to 4.80 cm, overlap of the directly traveling sine wave and the reflected sine wave occurs. Peak to peak amplitude of this composite signal is 2.40 divisions.

6 Peak to peak amplitude of the reflected signal is 1.37 divisions.

7 Direct measurement on the figure of the phase difference between points 5 and 6 is approximately 180°. Therefore, the peak to peak amplitude of the direct signal must be 3.77 divisions.

8 From Eq. [2], α is determined to be approximately 2.6 nepers/m.

The amplitude of the reflected signal is taken directly from the photograph without correction for the fact that the reflection coefficient at the rigid end is less than unity. However, use of the calculated value 0.92 for this coefficient does not change the value of α significantly (the correction is -0.06 nepers/m), since the uncertainty in α in several times greater.

The results of the measurements on the propellant rod, i.e., the Q's of $\lambda/4$ and $\lambda/2$ resonance and the α's obtained by pulse techniques, are shown in Fig. 5, where the Q's have been converted to α's using Eq. [1], with $v = 810$ m/sec. It is seen that the data are reasonably well represented by a straight line corresponding to a Q of 5.5, although a correction for phase velocity dispersion would make the line curve upward slightly at 14,000 cps.

The relatively small scatter of the data in Fig. 5 is to some extent fortuitous because, as Fig. 6 shows, the velocity is quite sensitive to the temperature, which was not under rigid control for all of the experiment. Moreover, at higher frequencies the lower signal-to-noise ratio and velocity dispersion effects made for some uncertainty in the amplitude of the reflected signal. On the other hand, the large positive deviation of the data point at 3 kc/sec may be a real effect, since it is the average of four runs with small individual scatter.

No significant change in the Q for $\lambda/4$ resonance was detectable over the temperature range 55° to 75°F. At constant temperature, both Q and sound velocity as determined by $\lambda/2$ resonance experiments are slightly, but perceptibly, decreased by increasing the static pressure to 1000 psi. In Fig. 5, the triangular data points 1, 3, 2 represent the attenuation and resonant frequency before, during, and after pressurizing, respectively.

Few generalities can be drawn between the experiments performed here on this "linear, partly crystalline, highly polar, and generally not crosslinked" polymer (2) and those performed elsewhere on other polymers (9). However, the sensitivity of the velocity to changes in temperature and the insensitivity of the Q to changes in temperature are consistent with more detailed experiments reported for other polymers in the glassy state (9). Had the experiment on the propellant been extended to sufficiently high temperatures, a peak in the attenuation should have been observed in the general vicinity of the temperature corresponding to the transition to rubber-like consistency.

Phase Velocity and Young's Modulus

Bancroft (10) has calculated the retardation effect on the longitudinal velocity in rods in the form of a dependency upon the ratio of rod diameter d to sound wavelength λ and upon Poisson's ratio σ, which lies between 0 and 0.5. The effect will be of the order of 30% of the very low frequency velocity for a hypothetical value of $d/\lambda = 1.0$. However, in this experiment, with $d/\lambda = 0.31$ at 10,000 cps and Poisson's ratio estimated to be 0.4, the dispersion effect is about 4%, which is within the accuracy of the experiment. At 14,000 cps the dispersion is about 8%.

Another factor to be considered is that the acoustic attenuation acts to increase the phase velocity. However, as is shown in Appendix B, the magnitude of the increase for this experiment is about 1.3%, which is not significant here. Therefore, to within experimental error the effective Young's Modulus is

$$E = \rho c^2 \approx 1.7 \times (810 \text{ m/sec})^2 \approx 1.1 \times 10^{10} \text{ d/cm}^2 \quad [3]$$

Conclusions

The acoustic attenuation in cast double-base solid propellant rods has been measured by the successive use of $\lambda/4$ resonance, $\lambda/2$ resonance, and pulse techniques and shown to be reasonably well described by a Q of 5.5 in the frequency range 500 to 14,000 cps.

Over the temperature range 55° to 75°F, the velocity is observed from $\lambda/4$ resonance data to be a sensitive decreasing function of the temperature, with a temperature coefficient of -17 m/sec/°F, whereas the Q remains essentially constant. These results are consistent with more detailed experiments reported in the literature on other polymers in the glassy state.

An increase in pressure from 1 atm to 1000 psi produced a slight but detectable decrease in Q and in sound velocity.

Effects of α and of the ratio rod diameter/wavelength upon the sound velocity are shown to be negligible; thus, within experimental accuracy, one can use room temperature velocity, 810 m/sec, and the propellant density, 1.7 g/cm³, to get an effective Young's Modulus $\approx 1.1 \times 10^{10}$ d/cm².

Appendix A: Validity of the Approximation $Q = f_0/\Delta f$

The experimental quantities obtained from the resonant rods were the $f_0/\Delta f$'s that were treated as Q's in Eq. [6] in obtaining the corresponding values of α. The validity of this approximation can be checked by means of the driving end impedance expressions for the rods. For example, the impedance near resonance in the $\lambda/4$ rod is given by

$$z = \rho_0 v \coth\gamma l = \rho_0 v \frac{1 + j\tanh\alpha l \tan\beta l}{\tanh\alpha l + j\tan\beta l} \quad [A1]$$

where ρ_0 is rod density, l is rod length, and $\gamma = \alpha + j\beta$. At resonance, $\beta l = \pi/2$ and Eq. [3] reduces to

$$z = z_0 = \rho_0 v \tanh\alpha l \quad [A2]$$

At the two frequencies $f_0 \pm \Delta f/2$, corresponding to the half-power points, the impedance increases by a factor $2^{1/2}$ and is complex. Therefore

$$1.414 \tanh\alpha l \angle\theta_1 = \frac{1 + j\tanh\alpha l \tan\beta_1 l}{\tanh\alpha l + j\tan\beta_1 l} \quad [A3]$$

where θ_1 is the polar angle corresponding to β_1, and β_1 is obtained from the experimental value of $f_0/\Delta f$ which is 6.3 for $\lambda/4$ resonance. For $f_1 = (f_0 - \Delta f/2)$ and $l = \lambda_0/4$, one gets

$$\begin{aligned} \lambda_1 &= \lambda_0/0.921 \\ \beta_1 l &= 1.446 \\ \tan\beta_1 l &= 7.98 \end{aligned} \quad [A4]$$

which, when substituted into Eq. [A3], gives

$$\begin{aligned} \tanh\alpha l &\simeq 0.126 \\ \alpha &\simeq 0.333 \text{ nepers/m} \end{aligned} \quad [A5]$$

For the frequency $f_2 = (f_0 + \Delta f/2)$, one gets $\beta_2 l = 1.705$ and $\alpha \simeq 0.339$ nepers/m.

The value of α obtained from Eq. [1] is 0.35 nepers/m; hence, the approximation $Q = f_0/\Delta f$ contains an error small enough to be neglected here.

Appendix B: Effect of Attenuation on Phase Velocity

If viscoelastic damping is the main cause of attenuation, its effect upon the phase velocity can be obtained from the equation of motion of a rod using a complex Young's Modulus, $E^* = (E_1 + jE_2)$. The equation is given by

$$E^* \partial^2\eta/\partial z^2 = \rho(\partial^2\eta/\partial t^2)$$

where η is particle displacement in z direction. If $\eta = \eta e^{j\omega t - \gamma z}$, where $\gamma = \alpha + j\beta$, it can be shown, using Eq. [1] to substitute Q for α, that $E_1/E_2 = Q[1 - (1/4Q^2)]$ and

$$\left(\frac{E_1}{\rho}\right)^{1/2} = c = \frac{v}{\{1 - (1/4Q^2) + 1/Q^2[1 - (1/4Q^2)]\}^{1/2}} \quad [B1]$$

Substituting in an experimental value $Q = 5.5$, one gets

$$(E_1/\rho)^{1/2} = c = 0.987\, v \quad [B2]$$

where v, the phase velocity, is the experimentally measured rod velocity, and c would be the velocity if there were no losses.

Acknowledgment

The author wishes to acknowledge the assistance of Richard L. Hudson in selecting the pulse instrumentation.

References

1 McClure, F. T., Hart, R. W., and Bird, J. F. "Acoustic resonance in solid propellant rockets," J. Appl. Phys. **31**, 884–896 (1960).

2 Steinberger, R., "Preparation and properties of double-base propellants," *The Chemistry of Propellants* (Pergamon Press, Paris, 1960), pp. 246–264.

3 Landel, R. F. and Smith, T. L., "Viscoelastic properties of rubberlike composite propellants and filled elastomers," ARS J. **31**, 599–608 (1961).

4 Wegel, R. L. and Walther, H., "Internal dissipation in solids for small cyclic strains," J. Appl. Phys. **6**, 141–157 (1935).

5 Biesterfeldt, H. J., Lange, J. N., and Skudrzyk, E. J., "Vibrations of rods at frequencies below their radial resonance," J. Acoust. Soc. Am. **32**, 749–764 (1960).

6 Bronwell, A. B. and Beam, R. E., *Theory and Application of Microwaves* (McGraw-Hill Book Co. Inc., New York, 1947), 1st ed., pp. 178–180.

7 Roderick, R. L. and Truell, R., "The measurement of ultrasonic attenuation in solids and some results in steel," J. Appl. Phys. **23**, 267–279 (1952).

8 Skudrzyk, E., "Acoustic electronic pulse equipment," J. Acoust. Soc. Am. **32**, 565–571 (1960).

9 Woodward, A. E. and Sauer, J. A., "The dynamical properties of high polymers at low temperatures," *Advances in Polymer Science* (Springer-Verlag, Berlin, 1958–1960), Vol. 1, pp. 114–158.

10 Bancroft, D., "The velocity of longitudinal waves in cylindrical bars," Phys. Rev. **59**, 588–593 (1941).